ac Method of Factoring

Consider a polynomial expression of the form:

$ax^2 + bx + c$ or $ax^2 + bxy + cy^2$

The polynomial can be factored if there are two factors of *ac* whose sum is *b*.

There are two main situations.

One where the constant, *c*, is positive, $ax^2 + bx + c$ and
one when the constant, *c*, is negative $ax^2 + bx - c$.

When the constant is positive ...

The polynomial can be factored only if there are two factors of *ac* which <u>add</u> to be the absolute value of *b*.

When the constant is negative ...

The polynomial can be factored only if there are two factors of *ac* which have a <u>difference</u> of the absolute value of *b*.

We will ignore the sign of the middle number so that we don't have to keep saying the absolute value of *b* ... until the very end.

Then the FIRST question you have to answer is:

Are there two factors of __________ (*ac*) whose ____________ ("sum" or "difference" depending on *c*) is ______ (*b* without the sign)?

If your answer is yes - then the polynomial can be factored and the two factors you found which worked in answering the question will also work in the factoring.

Examples

1. $3+11a+6a^2$

The First Question:

Are there two factors of 3•6 = 18 whose sum (because the last number is positive) is 11 (the middle number) ? Yes, 2 and 9.

Rewrite the original problem and factor by grouping.

$3+11a+6a^2$

$= 3+9a+2a+6a^2$ (rewrite 11*a* as 9*a* + 2*a* largest first and same sign as the original middle number)

$= 3(1+3a)+2a(1+3a)$ (notice the common factor (1 + 3*a*))

$= (3+2a)(1+3a)$

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2. $2x^2+7x-15$

The First Question:

Are there two factors of 2•15 = 30 whose difference (because the last number is negative) is 7 (the middle number) ? Yes, 10 and 3.

Rewrite the original problem and factor by grouping.

$2x^2+7x-15$

$= 2x^2+10x-3x-15$ (rewrite 7x as 10x - 3x largest first and same sign as the original middle number)

$= 2x(x+5)-3(x+5)$ (notice the common factor (*x* + 5))

$= (x+5)(2x-3)$

Note: The last two examples say "Notice the common factor". This is **not a coincidence**! If you can answer yes to the question, it will factor in this method.

3. $3x^2 - 5x + 4$

The First Question:

Are there two factors of 3•4 = 12 whose sum (because the last number is positive) is 5 (the middle number - ignore the sign) ?

Factors of 12 are ...

- 1 and 12, sum = 13
- 2 and 6, sum = 8
- 3 and 4, sum = 7

Since there is not a factor pair whose sum is 5 the problem won't factor, write "prime".

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4. $600 - 800t - 800t^2$

Simplify the polynomial by factoring the Greatest Common Factor (GCF) first.

$$= 200\left(3 - 4t - 4t^2\right)$$

The First Question:

Are there two factors of 3•4 = 12 whose difference (because the last number is negative) is 4 (the middle number - ignore the sign) ? Yes, 6 and 2.

Rewrite the original problem and factor by grouping.

$$200\left(3 - 4t - 4t^2\right)$$

$$= 200\left(3 - 6t + 2t - 4t^2\right)$$ (rewrite 4*t* as –6*t* + 2*t* largest first and same sign as original middle number)

$$= 200\left(3\left(1 - 2t\right) + 2t\left(1 - 2t\right)\right)$$ (notice the common factor (1 – 2*t*))

$$= 200\left(3 + 2t\right)\left(1 - 2t\right)$$

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5. $300 + 400t - 400t^2$

Simplify the polynomial by factoring the Greatest Common Factor (GCF) first.

$= 100\left(3 + 4t - 4t^2\right)$

The First Question:

Are there two factors of 3•4 = 12 whose difference (because the last # is negative) is 4 (middle number - ignore the sign) ? Yes, 6 and 2.

Rewrite the original problem and factor by grouping.

$100\left(3 + 4t - 4t^2\right)$

$= 100\left(3 + 6t - 2t - 4t^2\right)$ (rewrite 4*t* as 6*t* – 2*t* largest first and same sign as original middle number)

$= 100\left(3(1 + 2t) - 2t(1 + 2t)\right)$ (notice the common factor (1 + 2*t*))

$= 100(3 - 2t)(1 + 2t)$

Exercises

For each trinomial below, write the complete process of factoring them. Use the steps given in the previous models as your guide.

	Trinomial	Factors
1.	$2x^2 - 9x + 9$	$(2x - 3)(x - 3)$
2.	$20y^2 + 38y + 12$	$2(5y + 2)(2y + 3)$
3.	$12a^2 + 9a - 30$	$3(4a - 5)(a + 2)$
4.	$4x^2 - 23x + 15$	$(4x - 3)(x - 5)$
5.	$3r^2 - 8r - 16$	$(3r + 4)(m - 4)$